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The Mixing Behaviour of non-Newtonian Fluids in a Stirred Tank

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Summary:

Mixing of non-Newtonian Fluids is an important task for product quality and yield in chemical process industry. This paper deals with CFD prediction of a laboratory scale reactor for a Newtonian and a non-Newtonian Fluid. As constitutive Equation for the non-Newtonian Fluid a generalized Newtonian fluid is used.

Prediction are carried out for a two stage Intermig stirrer for different speeds of operation. The resulted local velocity and shear rate fields are discussed and the integral power input will be presented in a power number versus Reynolds number diagram. To get the in mixing technology well known single curve for the power input a corrected Reynolds number is introduced.

Keywords:

Non-Newtonian Fluids, generalized Newtonian Fluid, velocity field, shear rates, power number, Reynolds number, corrected Reynolds number

1 Introduction

Mixing of non-Newtonian Fluids is an important task for product quality and yield in chemical process industry. A great experimental based knowledge exists about mixing processes of Newtonian, and non-Newtonian Fluids in stirred vessels. Experimental data base means, correlations of integral dimensionless numbers as Reynolds number, power number and flow number for a given mixer setup under different stages of operation.

The mixing behaviour of non-Newtonian fluids is strong influenced by the local flow filed. From this point of view, it is important for design and optimization of a mixing process, dealing with non-Newtonian Fluids, to have also information about the local behaviour of quantities as the flow filed and the shear rate. The Computational Fluid Dynamics (CFD) is a tool, which has the ability to deliver both integral and local data of the flow field and other important quantities like shear rate and energy dissipation inside the stirred vessel.

This paper deals with CFD prediction of a laboratory scale reactor for a Newtonian and a generalized Newtonian Fluid. The generalized Newtonian Fluid is used to predict a fluid with shear thinning behaviour as well as a fluid with shear thickening behaviour. To describe the shear dependent viscosity a Carreau model is used.

2 Governing Equation

The Governing equation for the description of a time dependent, incompressible, laminar flow filed /1/ in a stirred vessel are the continuity equation

$$\nabla v = 0$$

and the momentum equation

$$\frac{\partial(\rho\, v)}{\partial t} + \nabla \cdot (\rho\, v\, v) = -\nabla p + \nabla \cdot T + \rho g\ .$$

In this equation t is the time, v the velocity vector, p the pressure, ρ the density and g the gravity.

The constitutive equation of the extra stress tensor of the Newtonian Fluid has the form:

$$T = \eta\, D\ ,$$

with a constant dynamic viscosity η. For the non-Newtonian Fluid the constitutive equation is given as:

$$T = \eta\left(\dot{\gamma}\right) D\,,$$

with a shear rate dependent viscosity.

The generalized shear rate has the form:

$$\dot{\gamma} = \sqrt{\left[-4\, I_2(D)\right]}\,.$$

In this equation $I_2(D)$ represents the second invariant of the rate of deformation tensor

$$D = \frac{1}{2}\left[\nabla\, v + (\nabla\, v)^{+}\right].$$

The shear rate dependent viscosity of the non-Newtonian Fluid has been predicted using a Carreau function /2/

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + \left(\lambda \dot{\gamma}\right)^2\right]^{\left(\frac{m-1}{2}\right)}.$$

In this function represents η_0 the viscosity at zero shear rate, η_∞ the viscosity at infinite shear rate, λ a time constant and m die slope of the cure.

3 Computational Fluid Dynamics Model

The governing equations described above are solved time dependent in three space coordinates using the commercial CFD code Star CD based on the finite volume method.

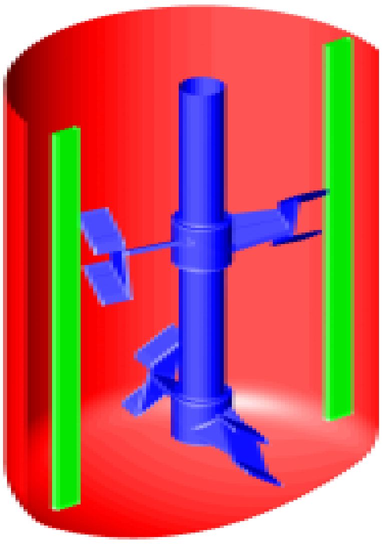


Figure 1: Vertical cut of the stirred vessel with two baffles

The convective fluxes of the momentum equation are approximated by 1st order upwind scheme and the PISO algorithm is used for pressure correction. The coupling between the inner rotating grid and the fixt outer grid is done by the Sliding Mesh method.

The geometry of the laboratory scaled stirred vessel is shown in figure 1. The vessel has a diameter of 0,26 m and a filling height 0f 0,32 m. As mixer two 90° staggered Intermig stirrer with 0,166 m diameter are modelled. The distance between the two Intermigs is 0,095 m and the distance of the bottom nearest Intermig to the cover is 0,07 m. Inside the vessel are two 180° staggered baffles with a height of 0,27 m, a width of 0,02 m and a thickness of 0,005 m. The CFD model has totally 321.600 hexahedron control volumes

Predictions are carried out for a Newtonian Fluid, a generalized Newtonian fluid with shear thinning behaviour and a generalized Newtonian fluid with shear thickening behaviour for speeds of revolution of 250 rpm, 500 rpm and 1000 rpm. The density for all fluids is 1000 kg/m^3 and the dynamic viscosity for the Newtonian case is 5 Pas.

The Carreau parameters for the shear thinning non-Newtonian fluid are chosen as: η_0 5 Pas, η_∞ 0,001 Pas, λ 0,1 s and m 0,2.

The parameters for the non-Newtoninan shear thickening fluid are: η_0 5 Pas, η_∞ 9,999 Pas, λ 0,1 s und m 0,2.

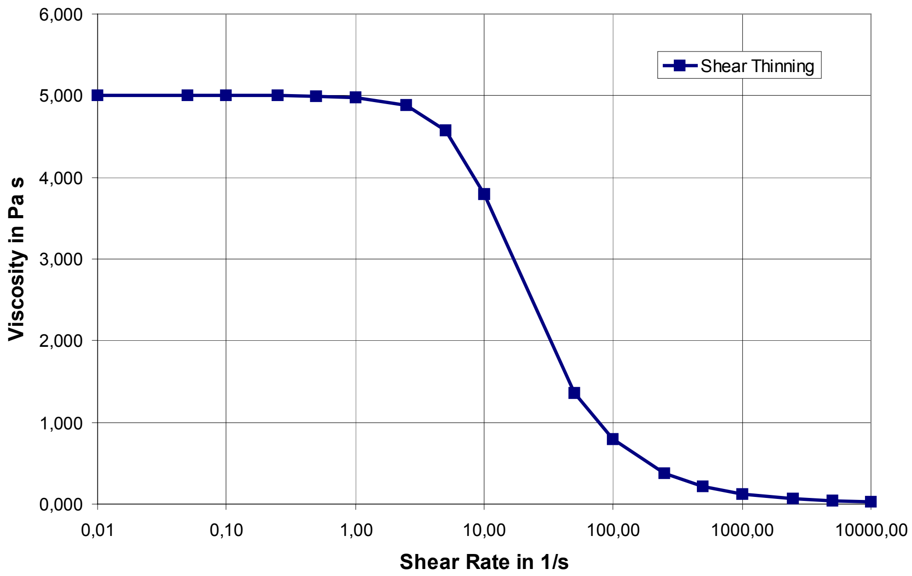


Figure 2: Non-Newtonian, shear thinning fluid, viscosity v.s. shear rate

The figures 2 and 3 show the shear viscosity predicted from the Carreau model as function of shear rate for the shear thinning and the shear thickening fluid. The Carreau parameter are chosen that way, that the significant change in viscosity for both fluids appears in the shear rate range of 1 1/s to 100 1/s. For real fluids the Carreau parameter are determined by adaptation to experimental data measured by viscometer.

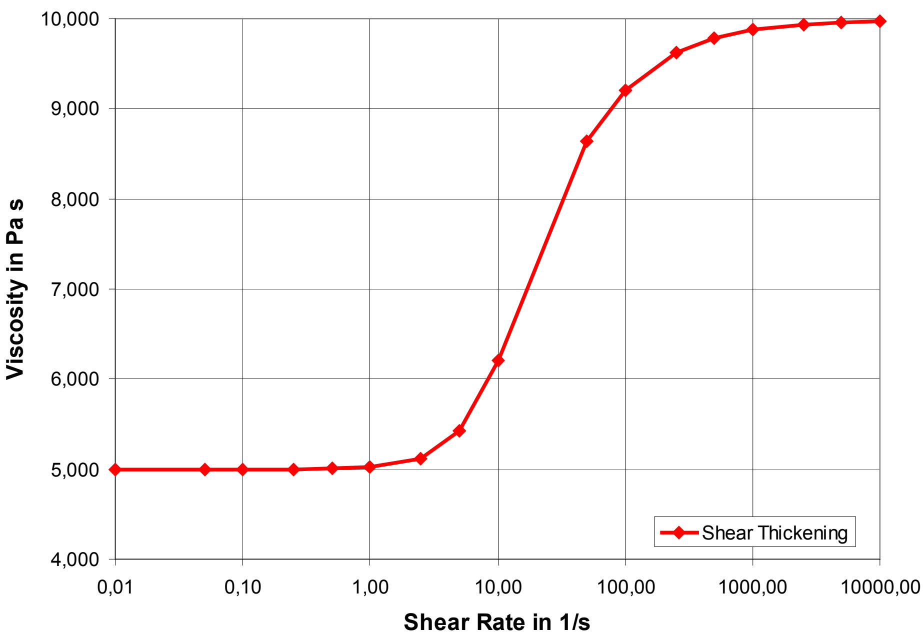
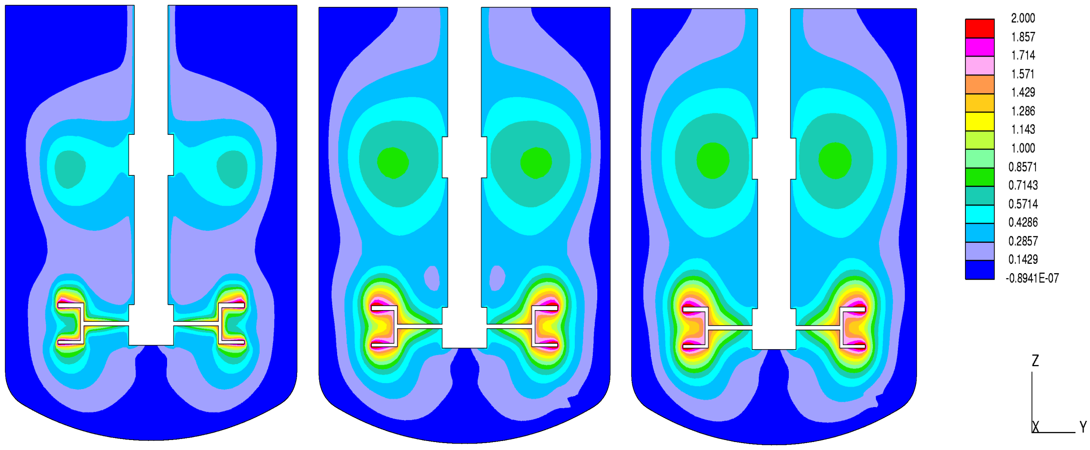


Figure 3: Non-Newtonian, shear thickening fluid, viscosity v.s. shear rate

4 Results

As a result of the predictions the velocity magnitude of the three different fluids for a speed of revolution of 250 rpm is shown in figure 4. The different pattern of the velocity magnitude plots suggests, that for the same speed of revolution, the power input of the stirrer is strong influenced by the local shear rate and viscosity.

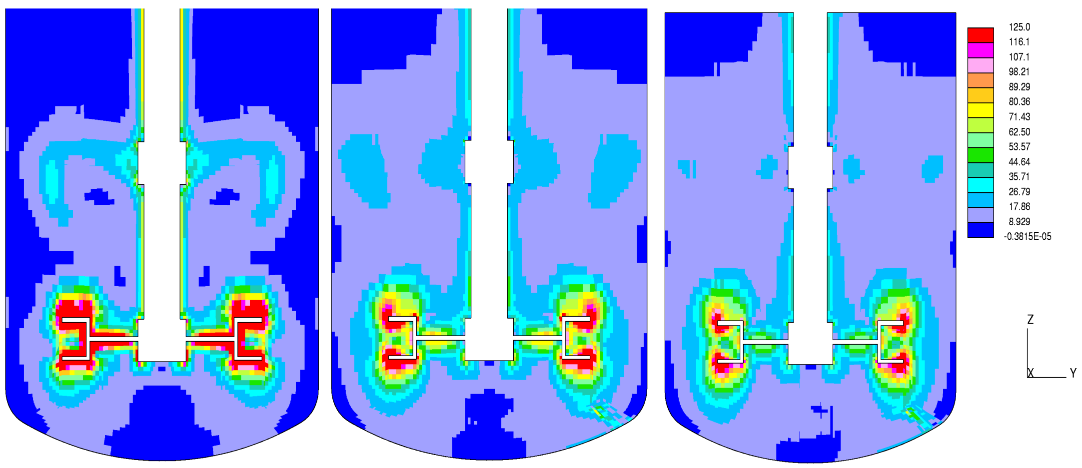


Shear thinning Newtonian Shear thickening

Figure 4: Magnitude of fluid velocity, speed of revolution 250 rpm

The momentum transport inside the stirred vessel takes place by convection and diffusion. The change of the extra stress tensor affected by the shear rate dependent decreasing or increasing viscosity – see figure 2 and 3 - is responsible for the different flow fields inside the vessel.

The pattern of the local shear rates for a speed of revolution of 250 rpm are given in figure 5. As expected the highest shear rates occur in the region of the stirrer. The integral shear rate over the stirrer surface decreases form the shear thinning to shear thickening fluid.



Shear thinning Newtonian Shear thickening

Figure 5: Local shear rate, speed of revolution 250 rpm

It is evident, that's also possible to predict integral quantities as the dimensionless power number.

$$Ne = \frac{P}{\rho\, n^3\, d^5}$$

from the local CFD data. The calculation of the power input

$$P = -\iint p\, u_j\; dS_j - \iint T_{ij}\, dS_j$$

will be done by integration over the forces acting at the surface of the stirrer and the shaft. (see figure 6).

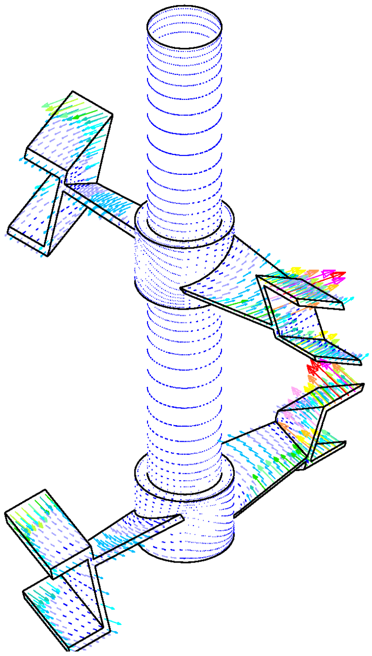


Figure 6: Plot of the resulting forces acting on surface of the stirrer and the shaft

The power input of the three investigated fluids for different Reynolds numbers is shown in figure 7. In this case the Reynolds number

$$\mathrm{Re} = \frac{n\, d^2\, \rho}{\eta_0}$$

is build with the stirrer diameter, the revolution frequency n, the density ρ and the zero viscosity η_0. The power number versus the Reynolds number plot in diagram 7 shows three separate curves for the power input. The lowest power input is given for the shear thinning fluid and the highest for the shear thickening fluid.

Using a corrected Reynolds number

$$\mathrm{Re}_C = \frac{n\, d^2\, \rho}{\eta\left(\dot{\gamma}_{\max}\right)}$$

build with the viscosity of highest shear rate at the stirrer, leads to the in mixing technology well known single curve for the power number v.s. Reynolds number diagram.

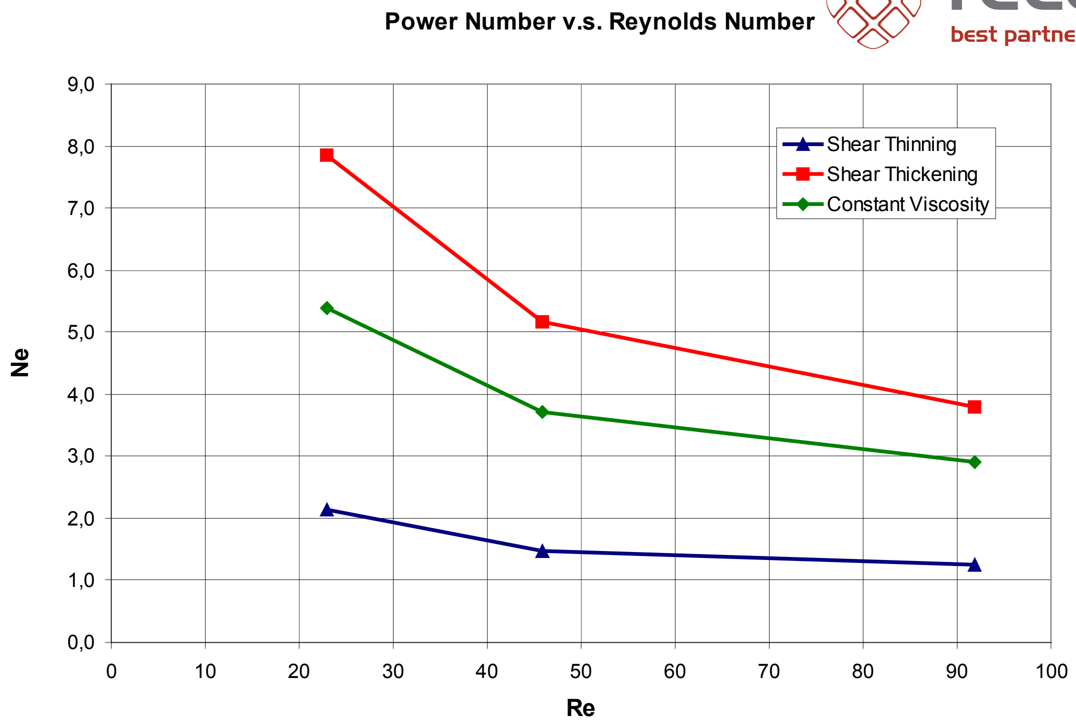


Figure 7: Power number v.s. Reynolds number

Figure 8 shows in a single curve the power number as function of Reynolds number of the investigated three different type of fluids. The corrected Reynolds numbers for the shear thinning fluid are 1004, 5093 and 20316. The flow filed for this case is turbulent or in the turbulent laminar transition region. The present predictions have been done for a laminar flow, this makes it apparent, that the highest derivations from the single curve occur for this case. The dashed line in figure 8 represents the power number 0,65 [3] of a two stage Intermig stirrer in the full turbulent flow regime.

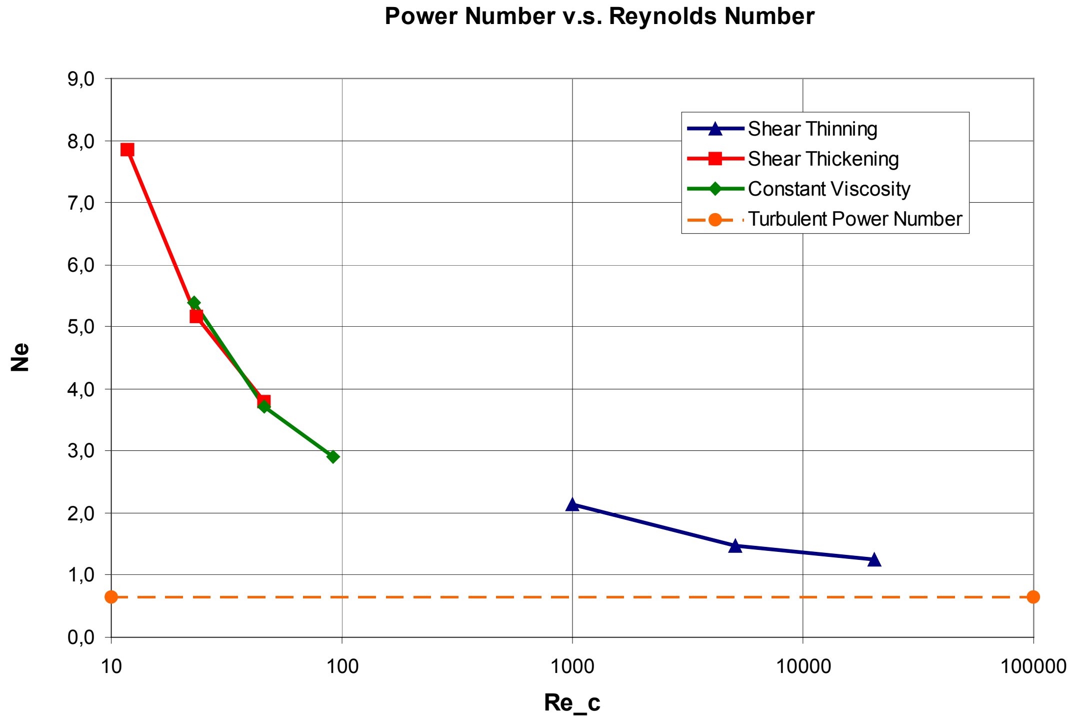


Figure 8: Power number v.s. corrected Reynolds number

5 Conclusion

Numerical predictions of the flow filed inside a stirred tank reactor of a Newtonian, a non-Newtonian shear thinning and a non-Newtonian shear thickening fluid are presented. For each of this fluids predictions are carried out for three different speeds of revolution.

The local velocity and shear rate fields for the 250 rpm revolution case are discussed and the influence of the shear dependent viscosity on the flow and shear rate fields are shown. The integral power input are predicted for different stages of operation via the force acting at the stirrer and the shaft.

Exemplary for integral quantities the power number versus Reynolds number diagram is presented for a Reynolds number build with zero viscosity and a corrected Reynolds number. To get the in mixing technology well known single curve for the power input a corrected Reynolds number is introduced. This Reynolds number is build with the viscosity at the highest shear rate occurring at the stirrer. To avoid a overestimation of singular data, occurring by shear rate prediction close to the stirrer surface, it seems to be useful to take a surface integrated average shear rate for the viscosity calculation.

6 References

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